**The van Hiele Model and its implication in blind students’ understanding in geometry**

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**Abstract**

The general research aim of this study was to investigate the appropriateness of the van Hiele model in promoting students with severe visual impairment’s understanding in geometry. The van Hiele model is a theory that seeks to explain how sighted children learn geometry that has also been used with students with vision impairment. Sixteen Greek secondary students with severe visual impairment participated in the present study. All participants were asked to explore by both hands a wide range of basic two- and three-dimensional geometric shapes. The outcomes of the study suggested that the van Hiele model a. may provide a reasonable structure for describing the geometrical learning of children with severe visual impairments, and b. may improve classroom practices when students’ understanding have been classified into levels.

**Introduction**

Blindness is associated with a range of geometry related conceptual problems. Fothergill (1980), for example, argues that it is a conceptual challenge for the blind child to infer the identity and substantiality of the object “out there” (p.55) using senses other than vision. Also, Lydon and Loretta (1973) highlight the difference between the sighted child’s developmental sequence of a concept as an uninterrupted cognitive motion from the parts to the whole, whereas for a blind child “once an object is out of physical grasp, it is gone”. According to Scholl (1986), the relation of parts to the whole can therefore be understood only if the blind student “plays with the parts”. Only through this process concepts of mental space and the grouping of objects can be perceived. Students with low vision also experience some of these challenges when learning geometry.

Shape perception by touch (or tactual shape perception) requires many factors to be considered. This is because there are multiple sources that provide information. Shapes differ in size, depth and composition and as a result provide different kinds of cues depending on the type of configuration. More analytically, there are at least six categories of tactual configuration (Millar, 1997). Millar (1994) refers to the recognition of shapes from a variety of stimuli such as: vibrotactile stimulation, flat and outline forms placed on the skin and in the hand, small three-dimensional shapes actively manipulated by hand, large three-dimensional objects which cannot be grasped in the hand, small continuous raised outlines and very small raised-dot patterns (e.g. braille).

It has been observed that the identification of geometric shapes by individuals who are blind is based mainly on the exploration of their angles and curves and not of their sides. Also, it seems that the characteristics of three-dimensional objects are learned easier and faster than those of two-dimensional ones (Ittyerah, 2010; Withagen et al., 2011). Finally, it seems that individuals with blindness prefer to use their indexes and thumbs when they actively manipulate objects (Heller, 1989).

Touch is an inter-sensory process. All receptors and cerebral areas must co-operate to create the outcome, tactual shape perception (Roberts & Wing, 2001).

The general research aim of this study was to investigate the appropriateness of the van Hiele model in students with vision impairment’s understanding in geometry. According to this model, the learner, assisted by appropriate instruction, passes through five levels beginning with recognition of shapes as a whole [level 0 – “Recognition”], progressing to discovery of the properties of figures and informal reasoning about these figures and their properties [levels 1 and 2 – “Analysis” and “Ordering” respectively], and culminating in a rigorous study of axiomatic geometry [levels 3 and 4 – “Deduction” and “Rigour” respectively) (Fuys, Geddes, & Tischler, 1988). The previous levels of understanding form a hierarchy and are mutually dependent and cumulative in character.

“The Van Hiele Model identifies five levels of thinking in geometry. According to this model, the learner, assisted by appropriate instructional experiences, passes through these levels beginning with recognition of shapes as a whole (level 0), progressing to discovery of the properties of figures and informal reasoning about these figures and their properties (levels 1 and 2), and culminating in a rigorous study of axiomatic geometry (levels 3 and 4)” (Fuys, Geddes, & Tischler, 1988: p1).

The research objectives were as follows: a. to assess and describe students with severe vision impairment’s levels of understanding while exploring small two- and three-dimensional manipulative geometric objects utilizing the van Hiele model, and b. to consider implications of teaching methods based on the utilization of the van Hiele model.

**Method**

### Participants

Sixteen secondary students from Greece with severe visual impairments (i.e. those who are blind and those with low vision) participated in the present study (age, M=16.94, SD=0.93). The teachers provided information about the visual acuities of the students, prognoses, and etiologies of the visual impairments. Of the 16 students, 10 were reported to have visual acuities of less than 20/200 and 6 had visual acuities between 20/100 and 20/200. The teachers did not provide information about the students’ visual fields and identified all the students as legally blind (with visual acuities of less than 20/200 or visual fields of less than 20 degrees). Finally, of the 16 students, none had additional disabilities, six had light perception only, eight had congenital conditions, and eight had adventitious conditions. (see Table 1).

<Please Insert Table 1 around here>

### Stimulus Material

Two- and three-dimensional geometric objects were used in this study, which could be actively manipulated by both hands by the participants. In total, 31 shapes were used, out of which 21 were two-dimensional (six triangles, 10 quadrilaterals, three polygons and two circles) and 10 three-dimensional ones (three pyramids, two cones, two spheres and three prisms). All shapes were constructed of plastic and each was unique.

**Research procedure**

The participants were asked to verbally describe the properties of the shapes in question while exploring them. All experiments were video-recorded and the camera shot focused exclusively on the participants’ hands. Permission for video recording was sought and granted from the students and their parents. A class was selected without disrupting students’ daily programme in class. Subjects were videotaped individually in 40 to 45-minute sessions. A preliminary phase took place before the main experiments. During this phase all participants had the chance to work out some shapes – which were not included in the main experiments – and comment on their attributes, texture and material. In turn, each participant was given a tray with all shapes and he/she randomly picked up one by one, without time limit and described the shape.

**Data analysis**

Data analysis was based on video recording sessions. In order to analyze the data linked to the first research objective, the researchers viewed the video multiple times and categorized all participants’ verbal explanations and descriptions according to the five levels in the mastery of geometry based on the van Hiele model of thinking.

The authors used the coding system of Fuys et al. (1988) to describe the quality of students’ responses as follows: a. students often did not respond consistently at the same level on a task, and the following codes were used in these situations: 0-1 indicates that the student’s understanding was in transition from level 0 to level 1, b. sometimes students were unable to respond to all the questions. In this case the omissions were indicated by a dash (--), c. when students had incomplete understanding of basic concepts (angle, curve or straight line) and also had little or no facility with the related terminology, then this was indicated by 0\*, and d. sometimes students responded spontaneously without even being asked questions or given instructions – for example, a student might have spontaneously described some properties of a shape. Other students needed a prompt; for example, “do you think the length of each side of a square is the same or not? or “what about the sides of a rectangle, are they equal or not?”. Frequently guidance was needed, for example, “can you count the corners for me?” or “Can you show me the sides and we can count them together”. The amount of the above during questioning was denoted by: s = spontaneous, p = prompt and g = guidance.

**Results**

Table 2 provides an approximate classification into van Hiele’s levels of understanding regarding two- and three-dimensional shapes in terms of a. their recognition (basic concepts), b. their properties, and c. their patterns.

**Basic Shapes**

It seemed that the participants’ level of understanding, according to Table 2, lies mostly within a transitional phase between levels 0 and 1 (33.13%). Also, Table 2 provided evidence of five students whose thinking or/understanding was categorised between level 1 and 2.

<Please Insert Table 2 around here>

Nevertheless, most of the participants seemed to be level 0 thinkers (see Table 3; total percentage response 50%). It seemed that participants were familiar with basic geometrical shapes such as circles, triangles, rectangles or squares but while trying to identify the set of the polygons they became confused and sometimes frustrated. Their thinking showed a failure to analyse the shapes in terms of their parts and properties (e.g. hexagon with no sides or trapezium with six or seven corners). They had memorised the names of some shapes without linking them with their corresponding properties. Also their grasp of mathematical terminology was poor and for the most part they remained at level 0 (Table 2).

The rest of the participants seemed to be in a phase of transition from level 0 to level 1 but mostly they remained at level 0. They could recognise a shape with ease and use the same method to identify shapes even though they did not know their names (e.g. octagon). It is worth mentioning that all students tried to analyse and work out the three-dimensional shapes as two-dimensional. None of them did remember the right terminology (especially for the prisms) and they were confused when they were counting corners. It seemed that they did not have a system of counting with stable reference points. Usually, they placed one of their hands on one of the corners of the three-dimensional shape to have a starting point to explore from and with the forefinger of the other hand they traced the corners of the shape going round and round.

### Properties

The thinking of the participants with respect to the properties of two dimensional shapes was almost uniformly at level 0 with an inclination to level 1 (56.25%, Table 3). The vast majority had an incomplete understanding of the terms “side” and “angle” and most times their recognition was on an “I felt it” basis rather than on the basis of the properties of the shapes in question. It could be argued that the participants’ approach was a holistic one based on their past experience without analysing the properties of the shapes and then draw a conclusion through this set of properties. Only three participants (I, M, and P, see Table 2) seemed to be very confident about the way they counted angles or sides. They made a clear distinction between the terms “sides” and “angles” but they preferred to use the term “points” instead of angles.

### Patterns

Regarding patterns of the two dimensional shapes, eight of the participants (A, D, G, J, K, L, N, and O, see Table 2) did not seem to be aware that every two-dimensional shape has the same number of sides and angles. Two participants (B and C) although they could figure out the pattern of the same number of sides and corners, they could not generalize.

Also, six participants (E, F, H, I, M, and P, see Table 2) did not face any problem with patterns; instead, after they became familiar with the procedure they started to give spontaneously all the information about a shape; for example, when they picked up the rectangle without the researcher’s intervention they started to give information about the name of the rectangle, the number of the sides, the number of the corners and mentioned the pattern of the same number. On the contrary, big confusion took place with all three-dimensional shapes apart from spheres.

<Please insert Table 3 around here>

#### An overview of the results

None of the participants reached any generalisation (they did not discover the pattern that the number of the sides of a two dimensional shape is always the same as the number of its angles) but participants M and P did reach this pattern; for this, they gave spontaneously answers which referred to the former logical scheme.

Below a synopsis is given in terms of the strategies used by the students for identifying the shapes in question: 1) Holding the shape in front of them (near their face or near their chest); marking an angle with a thumb and exploring the shape with the other hand while rotating it. This is actually an exploration of a two-dimensional shape in a three-dimensional way, and 2) Putting the shape firmly on the desk. This represents an exploration of a two-dimensional shape in a two-dimensional way. The desk may be considered as the Cartesian coordinate system and the shape could be recognised only from its two-dimensional properties.

**Discussion**

##### The first research objective of the present study referred to the applicability of van Hiele theory in describing blind students’ levels of understanding while exploring small two- and three-dimensional manipulative geometric objects. The analyses of the video-taped sessions provided rich information about the students’ levels of thinking. As described in Tables 2 and 3, a single level is not adequate to describe and classify students’ thinking. It could be suggested that a synthesis of levels might provide a more integrated “picture” of students’ understanding in geometry. Nevertheless, we argue that the van Hiele model provides a reasonable framework for describing the geometrical understanding of students with blindness and sheds light on students’ insights.

There is a consensus in most of the studies so far conducted that students tend to progress through a five-level sequence in a certain order and cannot master one level (Level n) if they have not mastered the previous one [(Level (n-1)] without instruction; when the opposite happened it was concluded that the student had performed only algorithmically on that higher level (Mayberry, 1983). An interesting point of view was expressed by a triad of researchers (Guitierrez, Jaime, & Fortuny, 1991) who distinguished “degrees of acquisition of a given level” (p. 238). They had noticed that many students’ answers are not entirely at a specific level but were laid out in a combination of consecutive levels (p. 237). In specific, they claimed that “the van Hiele levels are not discrete and we need to study in more depth the transition between levels” (p. 238).

In the present study, it seemed that most students’ showed a dominant level of thinking, with the presence of other levels as well. For this reason, the expression “this student is in transition between levels” (Burger & Shaughnessy 1986; Fuys et al., 1988; Usiskin 1982) fits in this study and also is often used by the researchers mentioned above.

Regarding the second research objective, it can be supported that the van Hiele model, provides a reasonable structure for describing the geometrical learning of children with visual impairments. The analyses of the video-taped sessions provided insight and valuable information: a. on students’ levels of thinking in geometry (Tables 2 & 3), and b. on factors which were affected students’ performances at different stages of the task. (e.g., vocabulary, tactile perception, misconceptions, learning styles - see Overview of the results).

The implications with respect to geometry may concern various aspects such as a. improvement of classroom practice when students’ understanding has been classified into levels, and b. teacher’s training in order to facilitate as much as possible students’ understanding using all he parameters of tactual shape perception (Millar, 1997).

Pegg (1985) praises the dynamic character of van Hiele’s model, stressing that:

The van Hiele theory is most useful for teachers in the advice it gives to planning for instruction. The fact that a pupil’s thinking can be classified at a particular level is not important itself. What is important are the implications to the teacher of the type of content and methods that are appropriate and the need to respond to pupils at *their* level of thinking. (p. 8)

In total, it is argued that individuals’ understanding does not fit neatly into one level or another (i.e. van Hiele’s levels). The use of “levels” satisfies the hierarchical structure of geometry or maths in general but it is not sufficiently refined to characterise thinking. This position can be justified by the following points: 1) the number of levels seem to be flexible, 2) performances generally seem to be spread across levels, and 3) performances are determined by what is taught. This implies that the nature of the van Hiele’s levels are more psychological than logical and undoubtedly has a bearing on teaching processes (Clements & Battista, 1992). That is to say, that progress from one level to the next is more dependent upon instruction than on age or biological maturation (Fuys et al., 1988). Tools such as the van Hiele model are useful to research on hierarchical thinking but more research is needed in terms of the reliability of the levels.

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Table 1: Characteristics of the participants

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Congenitally blind Adventitiously blind | | | | | | | | |
| Sex | agea | LP | Cause | Sex | agea | | LP | cause |
| F | 16 | No | PHPV | M | 19 | No | | Glaucoma |
| M | 17 | No | Optic nerve atrophy | M | 16 | No | | Glaucoma |
| F | 16 | No | Optic nerve atrophy | F | 17 | Yes | | Iridocyclitis |
| M | 17 | Yes | Congenital cataract | F | 17 | Yes | | Glaucoma |
| F | 16 | No | RP | F | 16 | No | | - |
| M | 16 | Yes | RP | F | 18 | Yes | | - |
| M | 18 | Yes | RLF | F | 17 | No | | - |
| M | 18 | No | RLF | M | 17 | No | | - |
| Note: F= female; M= male; LP= light perception; PHPV= persistent hyperplastic primary vitreous; RP = retinitis pigmentosa; RLF=retrolental fibroplasias.  a Age given in years. | | | | | | | | |

Table 2. Secondary Students’ Level of Thinking on 2-D and 3-D shapes.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **VAN HIELE LEVELS OF UNDERSTANDING** | | | | | | | | | | | | | | | |
|  | **PARTICIPANTS (N =16)** | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P |
| **CATEGORIES** | Shapes |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **Basic**  **Concepts** | Triangles | 0-1 | 0\* | 1-2 | 0-1 | 0-1 | 1-2 | 0-1 | 1-2 | 1-2 | 0-1 | 0-1 | 0-1 | 1-2 | 0\* | 0-1 | 1-2 |
| Parallelograms | 0\* | 0\* | 1-2 | 0-1 | 1-2 | 1-2 | 0-1 | 1-2 | 1-2 | 0\* | 0\* | 0-1 | 1-2 | 0\* | 0\* | 1-2 |
| Rectangles | 0-1 | 0-1 | 1-2 | 0-1 | 2\* | 2\* | 1-2 | 1-2 | 2 | 0-1 | 0-1 | 0-1 | 2\* | 0-1 | 0-1 | 2 |
| Rhombuses | 0\* | 0\* | 0-1 | 0\* | 0-1 | 1-2 | 0\* | 1-2 | 0\* | 0\* | 0\* | 0\* | 1-2 | 0\* | 0\* | 0\* |
| Squares | 0-1 | 0-1 | 1-2 | 0-1 | 2 | 2 | 1-2 | 1-2 | 2 | 0-1 | 0-1 | 0-1 | 2 | 0-1 | 0-1 | 2 |
| Trapeziums | 0\* | 0\* | 0-1 | 0\* | 0-1 | 0-1 | 0-1 | 0-1 | 0\* | 0\* | 0\* | 0\* | 1-2 | 0\* | 0\* | 0-1 |
| Pentagon | 0\* | 0-1 | 0-1 | 0-1 | 1-2 | 1-2 | 0-1 | 0-1 | 1-2 | 0-1 | 0-1 | 0\* | 1-2 | 0-1 | 0\* | 1-2 |
| Hexagon | 0\* | 0-1 | 0-1 | 0-1 | 1-2 | 1-2 | 0-1 | 0-1 | 1-2 | 0-1 | 0-1 | 0\* | 1-2 | 0-1 | 0\* | 1-2 |
| Octagon | -- | 0\* | -- | -- | 1-2 | 1-2 | -- | 0-1 | 1-2 | 0\* | 0-1 | -- | 0-1 | 0-1 | -- | 1-2 |
| Circles | 0-1 | 1-2 | 1-2 | 0-1 | 0-1 | 1-2 | 0-1 | 1-2 | 1-2 | 0-1 | 0-1 | 0-1 | 1-2 | 0-1 | 0-1 | 1-2 |
| Triangular Pyramid | -- | 0\* | 0\* | -- | 0-1 | 0 | -- | 0 | 0-1 | -- | -- | -- | 0-1 | 0-1 | -- | 1-2 |
| Square Pyramid | -- | 0-1 | 0-1 | -- | 0-1 | 0 | -- | 0 | 0-1 | -- | -- | -- | 0-1 | 0-1 | -- | 1-2 |
| Pentagonal Pyramid | -- | 0\* | 0\* | -- | 0-1 | 0 | -- | 0 | 0-1 | -- | -- | -- | 0-1 | 0-1 | -- | 1-2 |
| Sphere | 0-1 | 1-2 | 0-1 | 0-1 | 0-1 | 1-2 | 0-1 | 1-2 | 1-2 | 0-1 | 0-1 | 0-1 | 1-2 | 0-1 | 0-1 | 1-2 |
| Cone | -- | 0\* | 0\* | 0\* | 0\* | 0\* | 0-1 | 1-2 | 1-2 | 0\* | 0\* | 0-1 | 0-1 | 0-1 | 0-1 | 1-2 |
| Cube | 0-1 | 1-2 | 0-1 | 0-1 | 0-1 | 1-2 | 0-1 | 1-2 | 1-2 | -- | -- | -- | 0-1 | 0-1 | 0-1 | 1-2 |
| Cylinder | -- | 0\* | 0\* | 0\* | 0\* | 0\* | 0-1 | 1-2 | 1-2 | 0\* | 0\* | 0-1 | 0-1 | 0-1 | 0-1 | 1-2 |
| Triangular prism | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- |
| Square prism | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- |
| Pentagonal prism | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- | -- |
| **Properties** | Number of sides/number of angles | 0\* | 0-1g | 0-1s | 0-1g | 0-1p | 1-2s | 0-1g | 0-1g | 1-2g | 0-1g | 0\* | 0\* | 1-2p | 0-1g | 0\* | 1-2s |
| **Patterns** | Same number of sides and corners | -- | 0-1g | 0-1s | 0-1g | 0-1p | 0-1 | 0-1g | 0-1g | 1-2g | 0-1g | -- | -- | 1-2p | 0-1g | 0\* | 1-2s |

**Note**: Level 0 = Recognition/ Level 1 = Analysis/ Level 2 = Ordering/ Level 3 = Deduction/ Level 4 = Rigour

**Key**: (--) = Unable to respond/ 0\* = Weak response/ g = responded with guidance/ p = responded after prompting/ s = spontaneous response

Table 3. Percentage distribution of van Hiele’s levels of understanding for all participants (N = 16)

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **VAN HIELE LEVELS** | | | | | | | | |
|  | **0** | **0\*** | **0-1** | **1-2** | **2\*** | **2** | **3** | **4** | **--** |
| **Basic Concepts**  (2-D & 3-D Shapes) | 2.19% | 16.85% | 33.13% | 19.7% | 0.94% | 2.19% | 0% | 0% | 25% |
| **Properties**  Counting the number of sides/number of angles (only for 2-D Shapes) | 0% | 18.75% | 56.25% | 25% | 0% | 0% | 0% | 0% | 0% |
| **Patterns**  Compare the number of sides and corners (only for 2-D Shapes) | 0% | 6.25% | 56.25% | 18.75% | 0% | 0% | 0% | 0% | 18.75% |

**Key**: (--) = Unable to respond/ 0\* = Weak response/ g = responded with guidance/ p = responded after prompting/ s = spontaneous response